

الاسم:  
الرقم:

مسابقة في مادة الفيزياء  
المدة: 3 ساعات

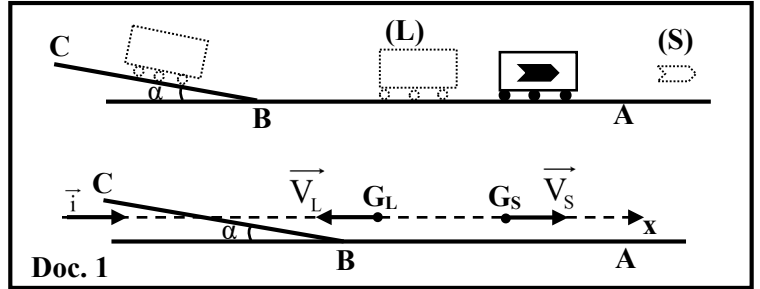
**This exam is formed of four obligatory exercises in four pages.**  
**The use of non-programmable calculator is recommended.**

### Exercise 1 (7 pts)

#### Recoil of a launcher

A launcher (L) with a mass  $M = 4000$  kg carries an object (S) of mass  $m = 100$  kg.

The system [(L), (S)] is initially at rest on a horizontal track [AB]. During the launch, the object is fired horizontally, and the launcher recoils backward.  $G_L$  and  $G_S$  are the centers of mass of (L) and (S) respectively. The x-axis is horizontal, passes through  $G_L$  and  $G_S$ , and is oriented along the unit vector  $\vec{i}$  (Doc. 1). The aim of this exercise is to study the motion of the launcher after launching the object.

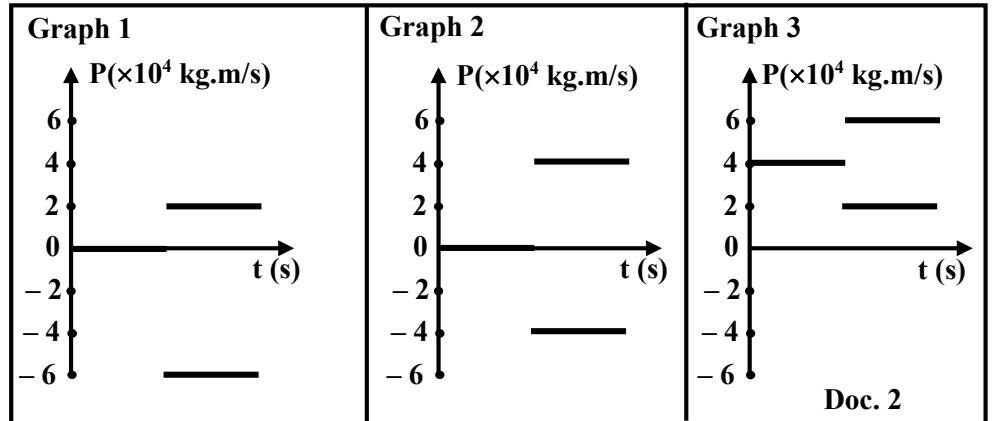


Take:

- The horizontal plane containing the x-axis as the reference level for gravitational potential energy;
- $g = 10 \text{ m/s}^2$ .

#### 1) Launching of the object

The graphs in document 2 represent, as a function of time, the possible algebraic values of the linear momentum of  $G_L$  and  $G_S$  before and after the launch.



Doc. 2

- 1.1) Two of the three graphs in document 2, do not describe correctly the algebraic values of the linear momentum of  $G_L$  and  $G_S$  before and after the launch. Specify which ones.

- 1.2) Deduce that the velocities  $\vec{V}_L$  of  $G_L$  and  $\vec{V}_S$  of  $G_S$  just after launch are respectively:

$$\vec{V}_L = -10 \vec{i} \text{ (m/s)} \text{ and } \vec{V}_S = 400 \vec{i} \text{ (m/s)}.$$

#### 2) Motion of the launcher

After the launch, (L) recoils along AB and then ascends an inclined plane BC that makes an angle  $\alpha$  with the horizontal, where  $\sin \alpha = 0.2$ .

Along AB, the motion of (L) is frictionless; along BC, (L) is subjected to a constant friction force  $\vec{f}$  parallel to the direction of motion.

- 2.1) By applying Newton's second law on (L) between points A and B, show that the motion of (L) between these two points is uniform.

- 2.2) At an instant  $t_0 = 0$ , taken as an initial time, (L) reaches the inclined plane BC at point B with the velocity  $\vec{V}_L$  and then reaches the highest point at an instant  $t_4$ .

The adjacent table gives, between the instants  $t_0 = 0$  and  $t_4$ , some values ( $E_1$  and  $E_2$ ) of the kinetic energy of (L) and of the gravitational potential energy of the system [(L) – Earth].

t	$t_0 = 0$	$t_1$	$t_2$	$t_3$	$t_4$
$E_1$ ( $\times 10^4$ J)	0	1	2	3	4
$E_2$ ( $\times 10^4$ J)	20	14	9	4	0

- 2.2.1) Specify which of the energies  $E_1$  and  $E_2$  corresponds to the kinetic energy KE of (L) and which corresponds to the gravitational potential energy GPE of the system [(L) – Earth].
- 2.2.2) Deduce the maximum distance « d » traveled by (L) along BC.
- 2.2.3) Determine the variation of the mechanical energy of the system [(L) – Earth] between  $t_0 = 0$  and  $t_4$ .
- 2.2.4) Deduce the magnitude  $f$  of  $\vec{f}$ .

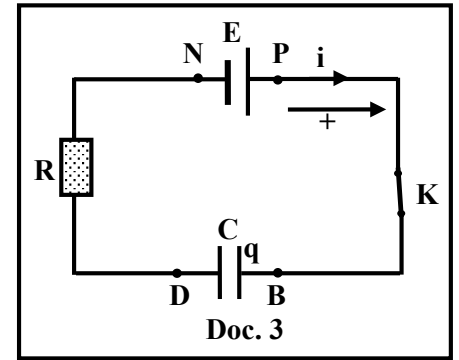
## Exercise 2 (7 pts)

### Capacitance of a capacitor

The aim of this exercise is to determine the capacitance  $C$  of a capacitor by two methods.

We build the series circuit shown in document 3, consisting of:

- an ideal battery of electromotive force  $E$ ;
- a resistor of resistance  $R = 2 \text{ k}\Omega$ ;
- a capacitor, initially uncharged, of capacitance  $C$ ;
- a switch  $K$ .



#### 1) First method

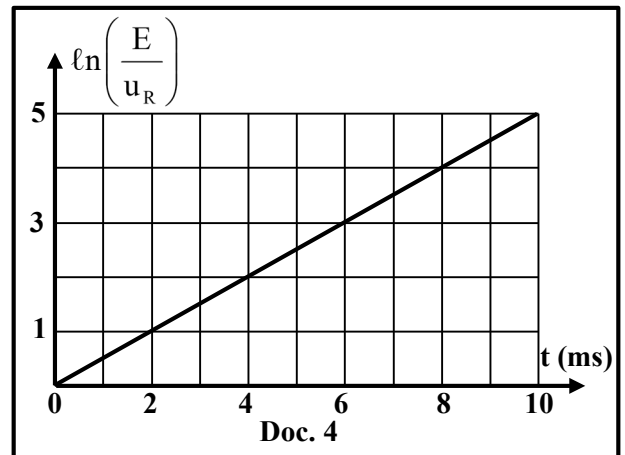
At  $t_0 = 0$ ,  $K$  is closed. At an instant  $t$ , the plate  $B$  of the capacitor carries a charge  $q$  and the circuit carries a current  $i$ .

- 1.1) Write the expression of  $i$  in terms of  $C$  and  $u_C$ , where  $u_C = u_{BD}$  is the voltage across the capacitor.
- 1.2) Determine the differential equation that describes the variation of the voltage  $u_C$ .
- 1.3) The solution of the obtained differential equation has the form:  $u_C = a + b e^{\alpha t}$  where  $a$ ,  $b$  and  $\alpha$  are constants. Determine the expressions of  $a$ ,  $b$  and  $\alpha$  in terms of  $E$ ,  $R$  and  $C$ .
- 1.4) Deduce the expression of the voltage  $u_R = u_{DN}$  across the resistor in terms of  $E$ ,  $R$ ,  $C$  and  $t$ .

1.5) Show that:  $\ln\left(\frac{E}{u_R}\right) = A \times t$ , where  $A$  is a constant to be determined in terms of  $R$  and  $C$ .

1.6) The curve of document 4, shows  $\ln\left(\frac{E}{u_R}\right)$  as a function of time. Show that the shape of this curve is in agreement with the equation obtained in part 1.5.

1.7) Using document 4, determine the value of  $C$ .



#### 2) Second method

We replace the capacitor of capacitance  $C$  by another capacitor of capacitance  $C' = 4 \mu\text{F}$  and we charge it under a voltage  $E' = \frac{E}{2}$ .

At the end of the charging process, the electric energy stored in the capacitor is  $W'$ .

- 2.1) Write, in terms of  $C'$  and  $E'$ , the expression of the electric energy  $W'$ .
- 2.2) Determine again the value of  $C$ , such that  $W' = W$ , where  $W$  is the electric energy stored in the capacitor of capacitance  $C$  at the end of charging under the voltage  $E$ .

### Exercise 3 (6.5 pts)

#### Wireless charger

The aim of this exercise is to study the phenomenon of electromagnetic induction in a wireless charger system.

#### 1) Electromagnetic induction

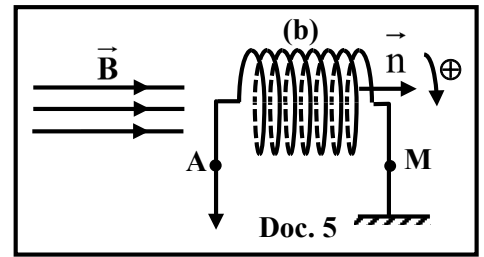
Consider a flat circular coil (b), with  $N = 1000$  turns and radius  $r = 2$  cm.

The two ends, A and M, of coil (b) are connected to an oscilloscope.

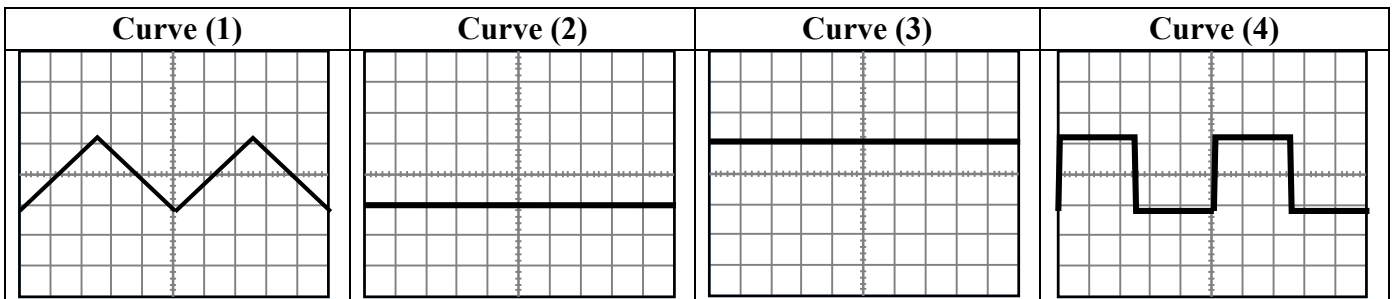
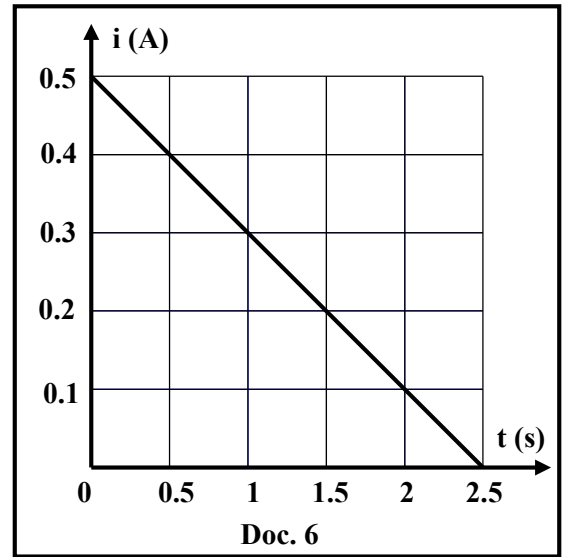
The axis of (b) is horizontal. The normal  $\vec{n}$  to the plane of the turns of (b) is oriented as shown in document 5. A uniform magnetic field  $\vec{B}$ , parallel to the axis of (b), is created by an electric current  $i$ .

The value of  $\vec{B}$  is  $B = 4 \times 10^{-3} i$  (SI).

The curve of document 6, shows  $i$  as a function of time.



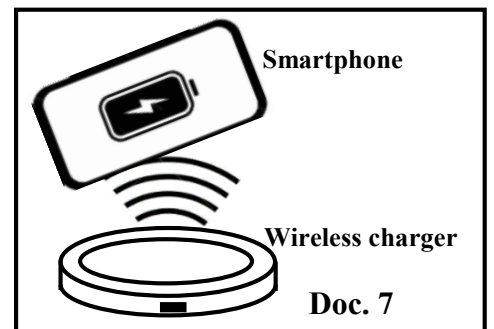
- 1.1) Justify the following statement: when  $\vec{B}$  and  $\vec{n}$  are parallel and have the same direction, the magnetic flux crosses (b) is maximum.
- 1.2) Using document 6, determine the expression of  $B$  in terms of  $t$ .
- 1.3) Taking into consideration the positive direction indicated on document 5, determine in terms of  $t$ , the expression of the magnetic flux that crosses (b).
- 1.4) Deduce the value of the induced electromotive force «  $e$  » in (b) during the time interval  $[0 ; 2.5$  s].
- 1.5) Why there is no induced current in (b)?
- 1.6) Knowing that (b) is oriented positively from A to M, choose with justification the curve among the four below curves that will be displayed on the screen of the oscilloscope.



#### 2) Functioning of a wireless charger

Wireless charging is based on the principle of electromagnetic induction. A variable alternating current flowing through a first coil in the wireless charger, generates a variable magnetic field. When a second coil, in a compatible smartphone is crossed by this field, a voltage is induced, thus charging the battery of the smartphone (Doc. 7).

- 2.1) During the wireless charging, indicate the coil that acts as the inducing source and the one that acts as the induced circuit.



- 2.2) The total power received by the wireless charger is

$P_{\text{received}} = 20$  W and the useful power transmitted to the battery of the smartphone is  $P_{\text{useful}} = 12$  W.

Calculate the efficiency  $r$  of charging, knowing that  $r = \frac{P_{\text{useful}}}{P_{\text{received}}}$ .

## Exercise 4 (7 pts)

### Hydrogen and helium atoms

The aim of this exercise is to study the absorption spectrum of the hydrogen atom and the helium atom in order to compare the surface temperature of two stars.

Given:

- Planck's constant:  $h = 6.62 \times 10^{-34}$  J.s;
- $1 \text{ eV} = 1.6 \times 10^{-19}$  J;
- Speed of light in vacuum:  $c = 3 \times 10^8$  m/s;
- The wavelength in vacuum of the visible spectrum:  $400 \text{ nm} \leq \lambda \leq 800 \text{ nm}$ .

#### 1) Hydrogen atom

The energy of level  $n$  of the hydrogen atom is given by the relation:

$$E_n = -\frac{E_0}{n^2}; E_n \text{ in eV, } E_0 = 13.6 \text{ eV and } n \text{ is non-zero integer number.}$$

1.1) The energy of the hydrogen atom is quantized. Justify.

1.2) A hydrogen atom, initially in the first excited state ( $n = 2$ ), receives a photon of energy  $E$  and wavelength  $\lambda_H$  in vacuum, it thus passes to the level  $p$  ( $p > 2$ ) of energy  $E_p$ .

1.2.1) Indicate to which domain (visible, infrared or ultraviolet) the absorbed photon belongs if  $p \leq 6$ .

1.2.2) Show that:  $\frac{1}{\lambda_H} = R \left( \frac{1}{4} - \frac{1}{p^2} \right)$ , where  $R$  is constant to be determined in terms of  $E_0$ ,  $h$  and  $c$ .

1.2.3) Calculate the value of  $R$  in SI units.

1.2.4) Using the relation of part 1.2.2, calculate the maximum value  $\lambda_H$  of a photon capable to ionize a hydrogen atom initially in the first excited state ( $n = 2$ ).

#### 2) Helium atom

Document 8 shows a simplified energy diagram of the ground level  $E_1$ , some excited states and the ionized state  $E_\infty = 0$  of a helium atom.

Each of the three transitions ( $T_1$ ,  $T_2$  and  $T_3$ ) shown on the diagram of document 8 corresponds to an absorption of a photon by the atom.

2.1) Calculate, for each transition ( $T_1$ ,  $T_2$  and  $T_3$ ), the energy in eV of the absorbed photon.

2.2) Determine, in nm, the wavelength of each of the absorbed photon.

2.3) Classify each absorbed radiation to the appropriate domain (visible, infrared or ultraviolet).

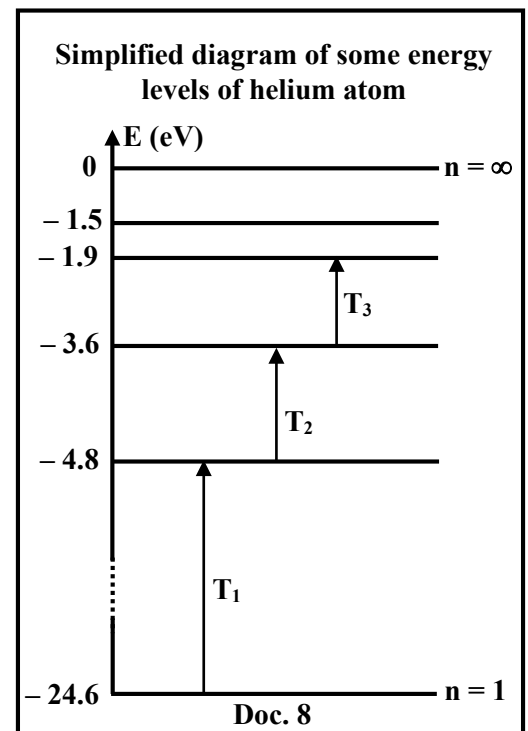
#### 3) Spectrum of a star and surface temperature

The analysis of the absorption spectrum of star «Sirius» shows mainly the presence of visible lines of the hydrogen atom. The analysis of the absorption spectrum of star «HD 144941» shows mainly the presence of lines corresponding to the transitions  $T_1$ ,  $T_2$  and  $T_3$  of the helium energy diagram (Doc. 8).

Knowing that the higher the ionization energy of an atom of the abundant element in the atmosphere of the star, the higher the star's surface temperature.

3.1) Define the ionization energy of an atom.

3.2) Specify which of the two stars, «Sirius» or «HD 144941», has the higher surface temperature.



Exercise 1 (7 pts) Recoil of a Launcher		
Part	Answer	Grade
1.1	Graph 1: incorrect <b>Justification:</b> The system [(L), (S)], initially at rest, is subjected to its weight and the force exerted by the support. The two forces cancel each other, and the system is isolated. Before launch, the system is at rest and its linear momentum is zero. During launch, the sum of the external forces remains zero; this system is still isolated, and its linear momentum is therefore conserved. $\vec{P}_{\text{just before}} = \vec{P}_{\text{just after}} ; \vec{0} = \vec{P}_{(L)} + \vec{P}_{(S)} ; \vec{P}_{(L)} = -\vec{P}_{(S)}$ which is not the case in this graph	0.5
	Graph 3 incorrect <b>Justification:</b> the system [(L), (S)], initially at rest, is subjected to its weight and the force exerted by the support. The two forces cancel each other, and the system is isolated. Before launch, the system is at rest and its linear momentum is zero, which is not the case in this graph. <b>Or</b> $\vec{P}_{\text{just after}} \neq \vec{0}$	0.5
1.2	From curve 2, the linear momentum of (L) and of (S) after launch: $\vec{P}_{(L)} = -\vec{P}_{(S)} = -4 \times 10^4 \vec{i}$ $\vec{P}_{(L)} = M \vec{V}_L = -4 \times 10^4 \vec{i} = 4000 \vec{V}_L ; \vec{V}_L = -10 \vec{i} \text{ (m/s)}$ $\vec{P}_{(S)} = m \vec{V}_S = 4 \times 10^4 \vec{i} = 100 \vec{V}_S ; \vec{V}_S = 400 \vec{i} \text{ (m/s)}$	1
2.1	Between A and B : $\frac{d\vec{P}}{dt} = \sum \vec{F}_{\text{ext}} = M\vec{g} + \vec{N} = \vec{0} ; \frac{d\vec{P}}{dt} = \vec{0} ; \vec{P} = \text{constant} = M \vec{V}$ Therefore, the velocities of the launcher is constant and its movement is uniform.	1
2.2.1	$E_1$ corresponds to the gravitational potential energy Since at $t_0 = 0$ , (L) is at B, on the reference level for gravitational potential energy Therefore $E_1 = 0 = \text{GPE}$ . $E_2$ corresponds to kinetic energy Since at $t_4$ , (L) reaches the highest point, therefore its speed on this point is zero. <b>Or</b> At $t_0 = 0$ , $E_2 = 20 \times 10^4 \text{ J}$ and $kE_C = \frac{1}{2} M V_L^2 = \frac{1}{2} \times 4000 \times 10^2 = 20 \times 10^4 \text{ J}$ Therefore $E_2 = KE_C$	0.5 0.5
	2.2.2	$\text{GPE}_{\text{max}} = 4 \times 10^4 \text{ J} = M g h_{\text{max}} = M g d \sin \alpha ; d = 5 \text{ m}$
2.2.3	$\Delta ME = ME_{t_4} - ME_{t_0} = 4 \times 10^4 - 20 \times 10^4 = -16 \times 10^4 \text{ J}$	1
2.2.4	$\Delta ME = W(\vec{f}) = f \times d \times \cos \pi = -f \times d$ $f = 32000 \text{ N}$	1

Exercise 2 (7 pts)		Capacitance of a capacitor	
Part	Answer		Grade
1.1	$i = \frac{dq}{dt}$ and $q = C u_C$ then $i = C \frac{du_C}{dt}$		0.25
1.2	Law of addition of volatges: $u_{PN} = u_{PB} + u_{BD} + u_{DN}$ $E = u_C + R i$ , but $i = C \frac{du_C}{dt}$ ; therefore : $E = R C \frac{du_C}{dt} + u_C$		0.25 0.5
1.3	$u_C = a + b e^{\alpha t}$ then $\frac{du_C}{dt} = b \alpha e^{\alpha t}$ we replace $u_C$ and $\frac{du_C}{dt}$ in the differential equation: $R C b \alpha e^{\alpha t} + a + b e^{\alpha t} = E$ ; $b e^{\alpha t} [R C \alpha + 1] + a = E$ ; This equality is verified for any t, by identification: $b e^{\alpha t} \neq 0$ then $a = E$ and $R C \alpha + 1 = 0$ therefore $\alpha = -\frac{1}{RC}$ $u_C = a + b e^{\alpha t}$ . but at $t = 0$ ; $u_C = 0$ then $b = -a = -E$ Therefore : $u_C = E (1 - e^{\alpha t})$ with $\alpha = -\frac{1}{RC}$		1.5
1.4	$u_R = u_{DN} = R i$ and $i = C \frac{du_C}{dt} = \frac{C E}{RC} e^{\alpha t} = \frac{E}{R} e^{\alpha t}$ then $u_R = R \frac{E}{R} e^{\alpha t} = E e^{\alpha t}$ with $\alpha = -\frac{1}{RC}$ or : $E = u_C + u_R$ then $u_R = E - u_C = E - E (1 - e^{\alpha t}) = E e^{\alpha t}$ with $\alpha = -\frac{1}{RC}$		1
1.5	$u_R = E e^{\alpha t}$ then $\frac{E}{u_R} = \frac{1}{e^{\alpha t}}$ ; therefore $\ln \frac{E}{u_R} = -\ln e^{\alpha t}$ Then $\ln \frac{E}{u_R} = -\alpha t$ , so $\ln \frac{E}{u_R} = \frac{1}{RC} \times t$ has the form $\ln \left( \frac{E}{u_R} \right) = A \times t$ then $A = \frac{1}{RC}$		1
1.6	The shape of the curve is an increasing straight line passing through the origin (positive slope). Which is in agreement with the expression : $\ln \left( \frac{E}{u_R} \right) = A \times t$ ; $A = \text{slope} = \frac{1}{RC}$		0.5
1.7	Slope = $\frac{5-0}{(10-0) \times 10^{-3}} = 500 \frac{1}{s}$ ; $500 = \frac{1}{RC}$ therefore $C = \frac{1}{500 \times 2000} = 1 \times 10^{-6} F = 1 \mu F$		0.25 0.5
2.1	At the end of capacitor charge $u_C = E$ then $W = \frac{1}{2} C E^2$ .		0.25
2.2	$W' = \frac{1}{2} C' E'^2$ and $W = \frac{1}{2} C E^2$ but $W = W'$ Then $\frac{1}{2} C' E'^2 = \frac{1}{2} C E^2$ therefore $C = C' \left( \frac{E'}{E} \right)^2 = C' \left( \frac{E}{2} \right)^2 = \frac{C'}{4} = \frac{4 \times 10^{-6}}{4} = 1 \times 10^{-6} F$ $C = 1 \mu F$		1

Exercise 3 (6.5 pts)		Wireless Charger
Part	Answer	Grade
1.1	The angle $\theta$ between the normal to the plane of (b) and the magnetic field is $\theta=0$ The magnetic flux $\Phi= N \cdot B \cdot S \cdot \cos\theta$ is maximum because N, B and S are constants and $\cos\theta = 1$ is maximum	1
1.2	$i(t)$ is a decreasing straight line not passing through the origin and its equation has the form $i = \text{slope} \times t + \text{constant}$ slope = $\frac{0-0.5}{2.5-0} = -0.2 \text{ A/s}$ and constant = 0.5 A (at $t=0$ ) then $i = -0.2 t + 0.5$ (S.I)  $B = 4 \times 10^{-3} \text{ T}$ $\Phi = 4 \times 10^{-3} i = 4 \times 10^{-3} (-0.2 t + 0.5) = -8 \times 10^{-4} t + 2 \times 10^{-3}$ (B in T and t in s)	1.5
1.3	$\Phi = NB \cdot S \cdot \cos\theta = 1000 \times (-8 \times 10^{-4} t + 2 \times 10^{-3}) \times \pi \times 0.02^2 = -10^{-3} t + 2.5 \times 10^{-3}$ (SI)	1
1.4	$e = -\frac{d\Phi}{dt} = 10^{-3} \text{ V}$	0.5
1.5	No current flows through the coil because it is connected to an oscilloscope with a very high-resistance	0.5
1.6	The oscilloscope allows us to observe the voltage $u_{AM}$ $u_{AM} = Ri - e$ but $i = 0$ then $u_{AM} = -e = -1 \times 10^{-3} \text{ V}$ So it's a constant but negative voltage, which corresponds to curve 2	1
2.1	Inducing source: first coil Induced circuit: second coil	0.25 0.25
2.2	$r = \frac{P_{\text{useful}}}{P_{\text{received}}} = \frac{12}{20} = 0.6 = 60 \%$	0.5

Exercise 4 (7 pts) Hydrogen and helium atom		
Part	Answer	Grade
1.1	$E_n$ has a specific value or $E_n$ takes discrete values .	0.5
1.2.1	Visible domain	0.25
1.2.2	When a hydrogen atom passes from a level $n = 2$ to a higher level $p$ , it absorbs a photon of energy : $h\nu = \frac{hc}{\lambda_H} = E_p - E_2 = -\frac{E_0}{p^2} + \frac{E_0}{2^2}$ So $\frac{1}{\lambda_H} = \frac{E_0}{hc} \left( \frac{1}{4} - \frac{1}{p^2} \right)$ has the form: $\frac{1}{\lambda_H} = R \left( \frac{1}{4} - \frac{1}{p^2} \right)$ with $R = \frac{E_0}{hc}$	1
1.2.3	$R = \frac{E_0}{hc} = \frac{13.6 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 1.0956 \times 10^7 \text{ m}^{-1}$	0.5
1.2.4	To ionize the atom means $p = \infty$ , So $\frac{1}{\lambda} = R \left( \frac{1}{4} - 0 \right)$ ; $\lambda_{H(\text{max})} = 3.6496 \times 10^{-7} \text{ m} \approx 365 \text{ nm}$	0.75
2.1	T <sub>1</sub> : $E_{\text{photon}} = -4.8 + 24.6 = 19.8 \text{ eV}$ T <sub>2</sub> : $E_{\text{photon}} = -3.6 + 4.8 = 1.2 \text{ eV}$ T <sub>3</sub> : $E_{\text{photon}} = -1.9 + 3.6 = 1.7 \text{ eV}$	0.25 0.25 0.25
2.2	T <sub>1</sub> : $E_{\text{photon}} = \frac{hc}{\lambda}$ ; $\lambda_1 = \frac{hc}{E_{\text{photon}}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{19.8 \times 1.6 \times 10^{-19}} = 6.268 \times 10^{-8} \text{ m} = 62.7 \text{ nm}$ T <sub>2</sub> : $\lambda_2 = 1.0343 \times 10^{-6} \text{ m} = 1034.3 \text{ nm}$ T <sub>3</sub> : $\lambda_3 = 7.301 \times 10^{-6} \text{ m} = 730.1 \text{ nm}$	0.5 0.25 0.25
2.3	T <sub>1</sub> → UV T <sub>2</sub> → IR T <sub>3</sub> → Visible	0.25 0.25 0.25
3.1	The ionization energy is the minimum energy needed to extract electron from an atom taken in its fundamental state.	0.5
3.2	Star « HD 144941 » , has the highest surface temperature. Because Ionization energy of the hydrogen atom = $0 - (-13.6) = 13.6 \text{ eV}$ Ionization energy of the helium atom = $0 - (-24.6) = 24.6 \text{ eV}$ $24.6 \text{ eV} > 13.6 \text{ eV}$	1