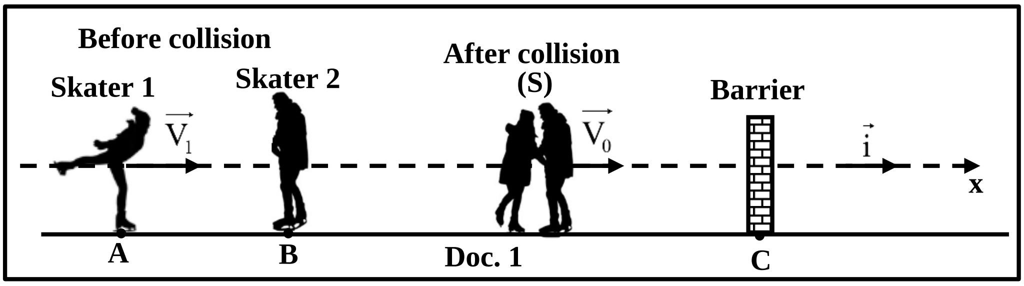
مسابقة في مادة الفيزياء — الاسم:
المدة: ساعتان — الرقم:

<u>This exam is formed of three obligatory exercises in three pages.</u>
<u>The use of non-programmable calculator is recommended.</u>

Exercise 1 (7 pts)

Collision between two ice skaters

Consider two skaters 1 and 2 of masses $m_1 = 60$ kg and $m_2 = 65$ kg respectively on a horizontal ice rink ABC. The aim of this exercise is to study the motion of the centers of masses of the two skaters along an x-axis with unit vector $\vec{i}$ (Doc. 1).



Doc. 1

Take the horizontal plane containing the x-axis as the reference level for gravitational potential energy.

1) Motion of skater 1

The center of mass of skater 1 moves between A and B with a constant velocity $\vec{V}_1 = V_1\,\vec{i}$.
Show that the forces of friction on skater 1 are negligible during its motion between A and B.

2) Collision between skater 1 and skater 2

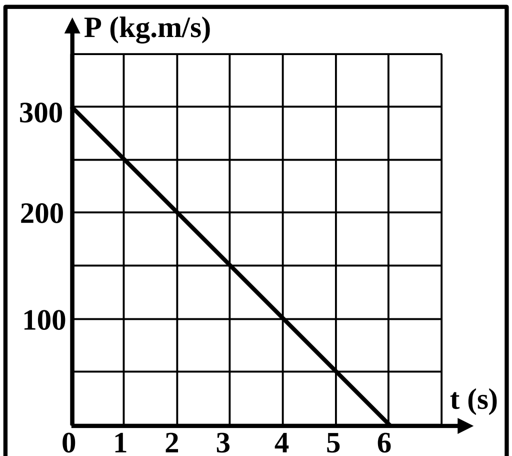
Skater 2 is initially at rest at point B. Skater 1 rushes towards skater 2. After collision, the two skaters hold together and form a single body (S).

At an instant $t_0 = 0$, the velocity of the center of mass of (S), just after collision, at B is $\vec{V}_0 = V_0\,\vec{i}$.

Determine V_0 in terms of V_1.

3) Motion of the center of mass of (S)

After collision, (S) stops at the instant t = 6 s due to the braking force $\vec{f}$ of constant magnitude f.
The curve in document 2, shows the algebraic value P of the linear momentum $\vec{P}$ of (S), as a function of time, during its motion between 0 and 6 s.



Doc. 2

3.1) Using document 2, show that $V_1 = 5$ m/s.

3.2) Show, by calculation, that the kinetic energy of the system (Skater 1, Skater 2) decreases during the collision.

3.3) Choose the correct answer.
The decrease in the kinetic energy of the system (Skater 1, Skater 2) during this collision is transformed into:
a) internal energy of the system (Skater 1, Skater 2, Atmosphere, Rink, Earth);
b) gravitational potential energy of the system (Skater 1, Skater 2, Earth);
c) internal energy of the system (Skater 1, Skater 2, Earth).

3.4) Referring to document 2, determine the expression of P in terms of time t between 0 and 6 s.

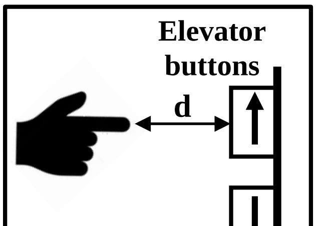
3.5) Determine f by applying Newton's second law on (S).

3.6) Deduce that (S) will stop before reaching the barrier at C, given that the distance BC = 12 m.

Exercise 2 (6.5 pts)

Capacitive proximity sensor

Capacitive proximity sensors are used in certain elevators, allowing a button to be activated without physical contact in order to limit the spread of bacteria and viruses (Doc. 3).



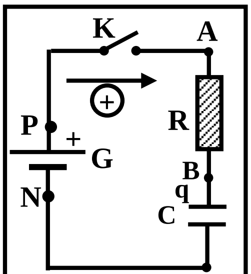
Doc. 3

The aim of this exercise is to determine the minimum capacitance of a capacitor in a capacitive proximity sensor used in elevator buttons to activate the sensor.

1) Capacitance of a capacitor

Document 4 shows a circuit contains in series:

- an ideal battery (G) of electromotive force E = 5 V;
- a resistor of large resistance R;
- a capacitor, initially uncharged, of capacitance C;
- a switch K.



Doc. 4

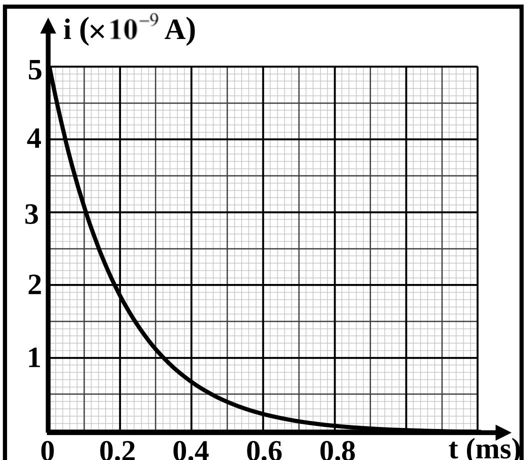
At instant $t_0 = 0$, K is closed and the charging process of the capacitor starts. At instant t, plate B of the capacitor carries a charge q and the circuit carries a current i.

1.1) Show that the differential equation that governs the variation of the voltage $u_{BM} = u_C$, across the capacitor is: $E = RC\frac{du_C}{dt} + u_C$.

1.2) The solution of the obtained differential equation has the form: $u_C = a - a\,e^{\frac{-t}{\tau}}$ where a and τ are constants. Determine the expressions of a and τ in terms of E, R and C.

1.3) Determine the expression of i, in terms of E, R, C and t.

1.4) The curve of document 5, shows i as a function of time. Show that the shape of this curve agrees with the expression found in part (1.3).



Doc. 5

1.5) Using document 5, determine:

1.5.1) the value of the resistance R;

1.5.2) the value of the time constant τ of the circuit.

1.6) Deduce that $C = 0.2 \times 10^{-12}$ F = 0.2 pF.

2) Capacitor in proximity sensor

A capacitor of capacitance C_d is used in the capacitive proximity sensor in elevator buttons. To activate this sensor, the finger must be within a certain distance d from the button (Doc. 3). The capacitance C_d varies with d according to the following relation:

$$C_d = 0.526 \times \frac{1}{d} \quad (C_d \text{ in pF ; d in cm and } d \neq 0).$$

The capacitive proximity sensor circuit is activated when $d \leq 2$ cm.

2.1) Determine, in pF, the minimum capacitance required to activate the sensor.

2.2) Document 4 is a simplified circuit used in capacitive proximity sensor. Deduce whether the capacitor of capacitance C = 0.2 pF may activate this sensor.

Exercise 3 (6.5 pts)

Hydrogen atom

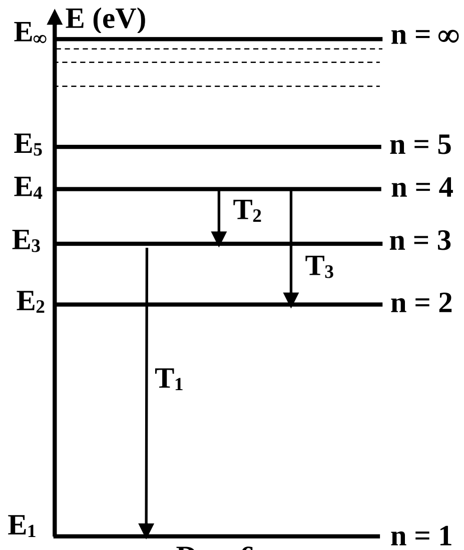
The energy levels of the hydrogen atom are given by:

$E_n = -\frac{E_0}{n^2}$; where E_0 = 13.6 eV and n is a whole non zero and positive number.

Document 6 shows a simplified diagram of the ground state E_1, the excited states E_2, E_3, E_4, E_5 and the ionized state $E_\infty = 0$ of the hydrogen atom.

Given:

- Planck's constant: $h = 6.63\times10^{-34}$ J·s;
- 1 eV = 1.6×10^{-19} J;
- visible spectrum: 400 nm < $\lambda_{visible}$ < 800 nm;
- 1 nm = 10^{-9} m;
- speed of light in air: $c = 3\times10^8$ m/s.



Doc. 6

1) Calculate E_1, E_2, E_3 and E_4 in eV.

2) The energy of the hydrogen atom is quantized. Justify.

3) The hydrogen atom found in its ground state, receives separately three photons (a), (b) and (c) whose energies are given in document 7.
Specify, for each photon, the final state of the atom chosen from the following three choices:

- ground state;
- third excited state;
- ionized state.

Photon	**Energy of the photon (eV)**
(a)	**14.20**
(b)	**12.75**
(c)	**3.40**

Doc. 7

4) The spectral lines of the hydrogen atom are grouped into series; each series corresponds to electronic transitions from an initial level n_i to a common final energy level n_f.
Consider the three following series:

- Lyman series: transitions from $n_i \geq 2$ to $n_f = 1$;
- Balmer series: transitions from $n_i \geq 3$ to $n_f = 2$;
- Paschen series: transitions from $n_i \geq 4$ to $n_f = 3$;

4.1) Justify that each of the transitions T_1, T_2 and T_3 shown in document 6 is accompanied by the emission of a photon.

4.2) Match each of the transitions T_1, T_2 and T_3 to one of the Lyman, Balmer or Paschen series.

4.3) For a given series (defined by $n_f = n$), the transition from $n_i = n + 1$ to $n_f = n$ corresponds to the largest emitted wavelength λ_{max} in this series. Explain.

4.4) Deduce that the transition T_2 is accompanied by the emission of a photon of maximum wavelength λ_{max} in its series.

4.5) Determine the value of λ_{max}.

4.6) To which range: visible, infrared or ultraviolet, does λ_{max} belong? Justify.

مسابقة في مادة الفيزياء
أسس التصحيح - إنكليزي

Exercise 1 (7 pts) **Collision between Two Ice Skaters**

Part	Answer	grade
1	**Method 1**: consider the system (Skater 1, Earth) $ME_A = KE_A + GPE_A = KE_A = \frac{1}{2} m_1 V_1^2$ $ME_B = KE_B + GPE_B = KE_B = \frac{1}{2} m_1 V_1^2$ Then $ME_A = ME_B$ = constant; so skater 1 slides without friction between A and B **Method 2**: consider the system (skater 1) According to the general relationship of Newton's second law: $\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext}$ since $\vec{P} = (m_1)\vec{v}_1$ = constant between A and B then $\sum \vec{F}_{ext} = \vec{0}$ Which corresponds $m\vec{g} + \vec{N} = \vec{0}$ therefore skater 1 slides without friction between A and B	**1**
2	$\vec{P}_{\text{before collision}} = \vec{P}_{\text{after collision}}$ $m_1 \overrightarrow{V_1} = (m_1 + m_2)\overrightarrow{V_0}$ then : $\mathbf{V_0} = \frac{\mathbf{m_1 V_1}}{\mathbf{m_1 + m_2}} = \frac{60}{125}\mathbf{V_1} = 0.48\mathbf{V_1}$	**1**
3.1	From the curve: $P_{\text{just after the collision}}$ = 300 kg.m/s $(m_1 + m_2)V_0 = 300$; $V_0 = \frac{300}{125} = 2.4$ m/s, Since $V_0 = 0.48 V_1$ then $V_1 = 5$ m/s	**0.75**
3.2	System (skater 1, skater 2) $KE_{\text{just before collision}} = \frac{1}{2} m_1 V_1^2 = 750$ J $KE_{\text{just after collision}} = \frac{1}{2}(m_1 + m_2) V_0^2 = 360$ J $< KE_{\text{just before collision}}$	**1**
3.3	a) internal energy of the system (Skater 1, Skater 2, Atmosphere, Rink, Earth)	**0.25**
3.4	The graph is a decreasing straight line; its equation is of the form: P = at + b At t = 0: P = 300, then b = 300 kg.m/s At t = 6 s: P = 0, then 0 = 6 a + 300 , so : a = – 50 kg.m/s^2, Then : **P = – 50 t + 300** with P in kg.m/s and t in second.	**1**
3.5	Applying Newton's second law on (S): $\Sigma \overrightarrow{F_{ext}} = \frac{d\vec{P}}{dt}$ then $m\vec{g} + \vec{N} + \vec{f} = \frac{d\vec{P}}{dt}$; But $m\vec{g} + \vec{N} = \vec{0}$ since the motion is horizontal then $-50\vec{i} = -f\vec{i}$ therefore **f = 50 N**	**1**
3.6	The variation in mechanical energy is equal to the work done by the frictional force. $\Delta ME = W_{\vec{f}}$ with $\Delta(ME) = ME_f - ME_0 = (KE_f + GPE_f) - (KE_0 + GPE_0)$ $\Delta ME = (KE_f) - (KE_0)$ $\Delta ME = \frac{1}{2} m (v_f^2 - v_0^2) = 0.5 \times 125 (0 - 2.4^2) = \mathbf{-360\ J}$ $\Delta ME = W_{\vec{f}}$ $-360 = -50 \times d$; d = 7.2 m < BC = 12 m. Therefore, (S) will stop before reaching the barrier.	**1**

Exercise 2 (6.5 pts)	Capacitive Proximity Sensor	
Part	**Answer**	**grade**
1.1	Law of addition of voltages: $u_{PN} = u_{PA} + u_{AB} + u_{BM} + u_{MN}$ $E = R\,i + u_C$, but $i = \frac{dq}{dt}$ and $q = C\,u_C$ then $i = C\,\frac{du_C}{dt}$; Therefore : $E = R\,C\,\frac{du_C}{dt} + u_C$	**1**
1.2	$u_C = a - a\,e^{\frac{-t}{\tau}}$; $\frac{du_C}{dt} = \frac{a}{\tau}\,e^{\frac{-t}{\tau}}$ we replace u_C and $\frac{du_C}{dt}$ in the differential equation : $E = RC\,\frac{a}{\tau}\,e^{\frac{-t}{\tau}} + a - a\,e^{\frac{-t}{\tau}}$; $a\,e^{\frac{-t}{\tau}}\,[\frac{RC}{\tau} - 1] + a = E$ This equality is verified for any t, by identification: $a\,e^{\frac{-t}{\tau}} \neq 0$ then **a = E** and $-\frac{RC}{\tau} + 1 = 0$ so $\boldsymbol{\tau = R\,C}$ therefore : $u_C = E\,(1 - e^{\frac{-t}{\tau}})$ with $\tau = RC$	**1**
1.3	$i = C\,\frac{du_C}{dt} = C\,\frac{E}{RC}\,e^{\frac{-t}{\tau}} = \frac{E}{R}\,e^{\frac{-t}{\tau}}$ **or** $i = \frac{E - u_C}{R} = \frac{E}{R}\,e^{\frac{-t}{\tau}}$	**1**
1.4	The curve agrees with the expression of i, since it decreases exponentially with time to zero	**0.5**
1.5.1	At t = 0, $i(0) = \frac{E}{R} = 5 \times 10^{-9}$ A ; $5 \times 10^{-9} = \frac{5}{R}$; $\mathbf{R = 10^9\ \Omega}$	**0.25** **0.5**
1.5.2	At $t = \tau$: $i = \frac{E}{R}\,(e^{-1}) = 0.37 \times 5 = 1.85 \times 10^{-9}$ **A** which corresponds graphically to $\boldsymbol{\tau = 0.2}$ **ms**	**0.5** **0.25**
1.6	$\tau = R \times C$; $C = \frac{0.2\times10^{-3}}{10^9} = 0.2 \times 10^{-12}$ F	**0.5**
2.1	$d \leq 2$ cm ; $\frac{1}{d} \geq \frac{1}{2}$ then $C_d \geq 0.526 \times \frac{1}{2} \geq 0.263$ pF ; therefore, $\mathbf{C_{min} = 0.263}$ **pF**	**0.5**
2.2	$C = 0.2 \times 10^{-12}$ F $= 0.2$ pF < 0.263 pF Consequently, the sensor is not activated.	**0.5**

Exercise 3 (6.5 pts)	**Hydrogen atom**	
Part	**Answer**	**grade**
1	n = 1 ; $E_1 = -\frac{E_0}{1^2}$ = **- 13.6 eV** ; n = 2 ; $E_2 = -\frac{E_0}{2^2}$ = **- 3.4 eV** n = 3 ; $E_3 = -\frac{E_0}{3^2}$ = **- 1.51 eV** ; n = 4 ; $E_4 = -\frac{E_0}{4^2}$ = **- 0.85 eV**	**1**
2	The values of En are not continuous or discrete. Each level has a specific energy; it takes discrete values depending on n.	**0.5**
3	see table below	**0.5** **0.5** **0.5**
4.1	Each transition occurs from a higher level n_i to a lower level n_f. This corresponds to a loss of energy in the atom, then it emits of a photon.	**0.5**
4.2	Transition T_1 : from $n_i = 3$ to $n_f = 1$ → **Lyman series** Transition T_2 : from $n_i = 4$ to $n_f = 3$ → **Paschen series** Transition T_3 : from $n_i = 4$ to $n_f = 2$ → **Balmer series**	**0.25** **0.25** **0.25**
4.3	The energy of the emitted photon is $E_{photon} = E_{n+1} - E_n$; $\frac{hc}{\lambda} = E_{n+1} - E_n$ Therefore, the smaller the energy difference, the larger the wavelength λ. Moreover $E_{n+1} - E_n$ is the smallest energy emitted in the series (since n_i = n +1 is the closest to n in each series). Therefore λ is maximum.	**0.5**
4.4	Because it is a transition between two consecutive levels n = 4 and n = 3 of the Paschen series.	**0.5**
4.5	$E_4 - E_3 = \frac{hc}{\lambda_{max}}$; $\lambda_{max} = \frac{hc}{E_4 - E_3}$; $\lambda_{max} = \frac{6.63\times10^{-34}\times3\times10^{8}}{(-0.85+1.51)\times1.6\times10^{-19}} = 1.884\times10^{-6}$ m = **1884 nm**	**0.75**
4.6	**Infrared** domain 1876 nm > 800 nm **or** it belongs to Paschen series.	**0.25** **0.25**

Part 3:

Photon	Energy of photon (eV)	Final state of the atom	Justification
a	14.20	ionized state	$E_{photon} > E_\infty - E_1$ = W ionisation = 13.6 eV
b	12.75	third excited state	$E_{photon} = E_4 - E_1$ = 12.75 eV The atom moves to the level n = 4, which corresponds to the third excited state.
c	3.40	ground state	-13.6 + 3.4 = - 10.2 ≠ En The atom does not absorb this photon; it therefore remains in the ground state.